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Theory of electromagnetic fluctuations for magnetized multi-species plasmas

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Analysis of electromagnetic fluctuations in plasma provides relevant information about the plasma state and its macroscopic properties. In particular, the solar wind persistently sustains a small but detectable level of magnetic fluctuation power even near thermal equilibrium. These fluctuations may be related to spontaneous electromagnetic fluctuations arising from the discreteness of charged particles. Here, we derive general expressions for the plasma fluctuations in a multi-species plasma following arbitrary distribution functions. This formalism, which generalizes and includes previous works on the subject, is then applied to the generation of electromagnetic fluctuations propagating along a background magnetic field in a plasma of two proton populations described by drifting bi-Maxwellians. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4894700]

I. INTRODUCTION

Plasmas often exhibit inherent electromagnetic fluctuations which are present even in the absence of plasma instabilities. The thermal motion of charged particles continuously produces scattered fluctuating waves, sometimes called *thermal noise*, which is then balanced by their reabsorption and dissipation. This process results in a finite and measurable level of magnetic fluctuations in the plasma. Although collisions in plasmas cannot generally ensure local thermodynamic equilibrium, in most cases the state of the system can be described by the linear response to the perturbations from a quasi-stable state via the fluctuation-dissipation theorem.[1]

The fundamentals of the theory of fluctuations have been rigorously discussed by Callen and Welton[1] and then applied to electromagnetic fluctuations in spatially dispersive media for the first time by Silin.[2] A detailed description for plasma fluctuations is given by Ichimaru[3] and Sitenko.[4] Since then, the study of the spectral properties of scattered electromagnetic fluctuations has provided substantial information about the plasma state in laboratory studies[5,6] and space-plasma measurements,[7–11] proving to be one of the most efficient methods for plasma diagnostics.

Recently, Schlickeiser and Yoon[12] derived general expressions for the electromagnetic fluctuation spectra in unmagnetized plasmas, which are valid for any relativistic and non-relativistic distribution function. Subsequent work applied this formalism mainly to the description of aperiodic (purely growing/damped) modes of a vast family of distribution functions.[12–16] Similarly, Lund *et al.*[17] derived expressions for the electrostatic fluctuations of a stable unmagnetized Maxwellian electron beam plasma. They showed that Debye shielding fluctuations are essentially unaffected by a low-density beam. However, the fluctuation level may be enhanced due to the presence of the beam and specially near marginal stability. Also, various normal modes may produce multiple peaks in the integrated spectra over frequencies or wavenumbers.

Araneda *et al.*[18] and Viñas *et al.*[19] studied magnetic fluctuations associated to Alfvén and whistler waves, respectively, in isotropic Maxwellian plasmas, and showed that heavily damped modes of the dispersion relation may play an important role in the emission and absorption of plasma fluctuations since they seem to constrain the structure of the fluctuation spectra. Navarro *et al.*[20] extended these ideas to anisotropic but quasi-stable bi-Maxwellian plasmas, showing that these fluctuations may be related to the fluctuating magnetic power observed in the solar wind below the linear instability thresholds.[21] Furthermore, the authors showed that the analytical description compared extremely well with hybrid simulations of protons and electrons. Certainly, other sources for observed fluctuations can be proposed. For instance, it is commonly accepted that enhanced fluctuations can appear in quasi-stable plasmas as a remnant of the earlier growth of plasma instabilities,[22,23] but this cannot explain the observed fluctuations far below the instability thresholds. On the other hand, fluctuations may be due to remnant MHD turbulent magnetic fluctuations advected from the Sun, which may be relevant if the time expansion of the MHD turbulence is much shorter than the fluctuation time scales. In this paper, we take the approach of considering the contribution of thermal fluctuations, which are known to arise naturally in most statistical systems, and particularly in quasi-equilibrium plasmas, without imposing initially unstable growing waves[17,20] or preexistent turbulence.

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The spontaneous electromagnetic fluctuations are completely determined by the macroscopic properties of the plasma and particularly by the distribution function of each species. Solar wind proton distributions are often observed to consist of an anisotropic Maxwellian-like core with temperatures $T_\perp$ and $T_\parallel$ with respect to the local background magnetic field $\mathbf{B}_0$.[24,25] Sometimes, the plasma also contains a secondary, closely isotropic, Maxwellian-like proton beam moving with an average drift velocity U_{bc} with respect to the proton core. Typically, U_{bc} varies between 0 and 1.5 times the local Alfvén speed v_A.[26–29] A similar feature can be observed for alpha-particle distributions.[30] Furthermore, in many situations, electron and ion distributions show non-thermal features which can be well represented by power-law distributions.[31–38] Thus, a complete description of electromagnetic fluctuations based on arbitrary distribution functions is certainly necessary.

In this work, we outline, in Sec. II, a derivation of electromagnetic fluctuations without the need of integral representations or form factors. This method extends the formalism described in the literature[4,12] to magnetized plasmas composed of an arbitrary number of species having different and arbitrary distribution functions. This description is useful for thermally anisotropic species and plasmas out of thermal equilibrium. In Sec. III, we apply the general formalism to electromagnetic fluctuations propagating along a background magnetic field for anisotropic bi-Maxwellian plasmas. We then analyze numerically these fluctuations for a three-component plasma consisting of a relatively dense proton core, a relatively tenuous proton beam, and electrons, in Sec. IV. Finally, in Sec. V we summarize and discuss the results.

II. ELECTROMAGNETIC FLUCTUATIONS IN MULTI-SPECIES PLASMAS

We will adapt, for multi-species plasmas, the statistical procedure described in Ref. 39. The classical statistical average of the fluctuating electric field component $E_j(t)$ over the phase-space x is given by

$$\langle E_j(t)\rangle = \frac{\sum_\alpha \int dx f_\alpha E_j}{\sum_\alpha \int dx f_\alpha}, \tag{1}$$

where $f_\alpha = f_\alpha(H)$ is the distribution function of species α, H is the total Hamiltonian of the system, and the sum is performed over all species. Suppose the system was prepared in its equilibrium state H_0, and then perturbed at $t=0$ so that $H = H_0 + \sum_\ell \Delta h_\ell$, where Δh_ℓ is the perturbing quantity associated to the $\ell = \{x, y, z\}$ direction.

Under these assumptions, f_α can be written to the first order in Δh_ℓ as

$$f_\alpha = F_\alpha + \sum_\ell \Delta h_\ell \frac{\partial F_\alpha}{\partial H_\ell}, \tag{2}$$

where $F_\alpha \equiv f_\alpha(H_0)$. Replacing Eq. (2) into (1) and collecting terms up to first order in Δh_ℓ, we obtain

$$\langle E_j(t)\rangle = \sum_\alpha \Bigg[\langle E_j(t)\rangle_\alpha + \sum_\ell \langle \Delta h_\ell(0) E_j(t)\rangle_\alpha^{(\ell)} - \langle E_j(t)\rangle_\alpha \sum_{\beta,\ell} \langle \Delta h_\ell(0)\rangle_\beta^{(\ell)} \Bigg], \tag{3}$$

where

$$\langle A(t)\rangle_\alpha = \frac{\int dx F_\alpha A}{\sum_\beta \int dx F_\beta}, \tag{4}$$

$$\langle A(t)\rangle_\alpha^{(\ell)} = \frac{\int dx (\partial F_\alpha/\partial H_\ell) A}{\sum_\beta \int dx F_\beta}. \tag{5}$$

Assuming that $\sum_\alpha \langle E_j(t)\rangle_\alpha = 0$ at quasi-equilibrium, then the Fourier spectrum of Eq. (3) is

$$\langle E_j(\mathbf{k},\omega)\rangle = i\omega \sum_{\alpha,\ell} \langle \Delta h_\ell(\mathbf{k},\omega) E_j^*(\mathbf{k},\omega)\rangle_\alpha^{(\ell)}, \tag{6}$$

where the asterisk represents a complex conjugate.

The dynamics of E_j is governed by the Maxwell equations, which can be written in Fourier-coordinates as[4]

$$\lambda_{ij}\langle E_j(\mathbf{k},\omega)\rangle = \frac{4\pi}{i\omega} J_i(\mathbf{k},\omega), \tag{7}$$

where $\lambda_{ij} = \lambda_{ij}(\mathbf{k},\omega)$ are the components of the dispersion tensor in vacuum, given by

$$\lambda_{ij} = \frac{c^2 k^2}{\omega^2}\left(\frac{k_i k_j}{k^2} - \delta_{ij}\right) + \delta_{ij}, \tag{8}$$

and J_i are the components of the total current density. In general, J_i consists of a spontaneous source current $\tilde{J}_i$ due to the particle motion in the plasma, and a current induced by the fluctuating electric field. In the linear approximation, this can be written as

$$J_i = \tilde{J}_i - i\omega \sum_\alpha \chi_{ij}^{(\alpha)} \langle E_j(\mathbf{k},\omega)\rangle, \tag{9}$$

where $\chi_{ij}^{(\alpha)}$ is the susceptibility tensor of species α, and the sum extends over all species.

Replacing Eq. (9) into Eq. (7), we obtain

$$\langle E_j(\mathbf{k},\omega)\rangle = \frac{4\pi}{i\omega} \Lambda_{jn}^{-1} \tilde{J}_n, \tag{10}$$

where Λ_{ij}^{-1} represents the inverse of the dispersion tensor $\Lambda_{ij} = \Lambda_{ij}(\mathbf{k},\omega)$, given by

$$\Lambda_{ij} = \lambda_{ij} + 4\pi \sum_\alpha \chi_{ij}^{(\alpha)}. \tag{11}$$

The perturbing quantity Δh_ℓ can be understood as the energy supplied by the source current $\tilde{J}_\ell$ to the wave, such that

$$\Delta h_\ell(\mathbf{k},\omega) = \frac{1}{2i\omega} \tilde{J}_\ell E_\ell(\mathbf{k},\omega). \tag{12}$$

Combining Eqs. (6), (10), and (12), we obtain

$$\sum_{\ell} \tilde{J}_{\ell}\left[\frac{8\pi}{i\omega}\Lambda_{j\ell}^{-1} - \sum_{\alpha}\langle E_{\ell}E_j^*\rangle_{\alpha}^{(\ell)}\right] = 0. \quad (13)$$

Since $\tilde{J}_{\ell}$ is randomly generated, then the term inside the parenthesis is zero for each ℓ. A similar condition is obtained for $\langle E_j E_{\ell}^*\rangle_{\alpha}^{(\ell)}$ in terms of $\Lambda_{j\ell}^{-1*}$. Writing $\Lambda_{j\ell}^{-1} = \Lambda_{jm}^{-1}\lambda_{mn}^{-1}\lambda_{n\ell}$ in Eq. (13) and replacing λ_{nm} from Eq. (11), we finally obtain

$$\begin{aligned}&\omega\langle E_iE_j^*\rangle_{\alpha}^{(i)} + \omega^*\langle E_iE_j^*\rangle_{\alpha}^{(j)}\\ &\quad = 32i\pi^2[\Lambda_{jm}^{-1}\chi_{mn}^{(\alpha)}\lambda_{ni}^{-1} - \lambda_{in}^{-1*}\chi_{nm}^{(\alpha)*}\Lambda_{mj}^{-1*}],\end{aligned} \quad (14)$$

where an implicit sum over the indexes m and n is performed. The total electric field fluctuations are given by

$$\langle E_iE_j^*\rangle = \sum_{\alpha}\langle E_iE_j^*\rangle_{\alpha}, \quad (15)$$

where $\langle E_iE_j^*\rangle_{\alpha}$ can be obtained from Eq. (14) if the distribution function F_{α} is given.

This Eq. (14), and the final desired result Eq. (15), are general enough that can be applied to non-Maxwellian distribution functions (e.g., with power-law tails, relativistic, and anisotropic distributions), and also for oblique propagation in magnetized plasmas, which we will present elsewhere shortly.

Equation (14) can be evaluated for frequencies where $\det(\Lambda_{ij}) \neq 0$. Let us note that $\det(\Lambda_{ij}) = 0$ is the dispersion relation which determines the frequencies $\omega = \omega(\mathbf{k})$ of the normal modes of the system. Hence, the fluctuation spectra have intense peaks near these modes.

The magnetic and density fluctuations can be derived through the use of Faraday equation and Gauss law in Maxwell's equations, from which we obtain

$$\langle B_iB_j^*\rangle = \varepsilon_{ilm}\varepsilon_{jns}\frac{c^2k_lk_n}{\omega^2}\langle E_mE_s^*\rangle, \quad (16)$$

$$\langle|\rho|^2\rangle = \frac{k_ik_j}{(4\pi)^2}\langle E_iE_j^*\rangle. \quad (17)$$

III. ELECTROMAGNETIC FLUCTUATIONS FOR BI-MAXWELLIAN PLASMAS

Let us considerer electromagnetic fluctuating waves propagating in a collisionless Vlasov-Maxwell plasma with a background magnetic field $\mathbf{B}_0 = B_0\hat{z}$, where each species component follows a bi-Maxwellian velocity distribution function (VDF)

$$F_{\alpha}(v_{\perp}, v_{\parallel}) = \frac{1}{\pi^{3/2}u_{\perp\alpha}^2u_{\parallel\alpha}}\exp\left[-\frac{v_{\perp}^2}{u_{\perp\alpha}^2} - \frac{(v_{\parallel}-U_{\alpha})^2}{u_{\parallel\alpha}^2}\right], \quad (18)$$

which is completely determined by the thermal speeds $u_{\parallel\alpha} = (2k_BT_{\parallel\alpha}/m_{\alpha})^{1/2}$ and $u_{\perp\alpha}^2 = (2k_BT_{\perp\alpha}/m_{\alpha})^{1/2}$, where $T_{\parallel\alpha}$ and $T_{\perp\alpha}$ are the parallel and perpendicular temperatures with respect to $\mathbf{B}_0$, and U_{α} is the drift along $\mathbf{B}_0$.

In this case, and from Eq. (5), it can be shown that

$$\langle E_iE_j^*\rangle_{\alpha}^{(\ell)} = -\frac{\langle E_iE_j^*\rangle_{\alpha}}{k_BT_{\ell\alpha}}, \quad (19)$$

where $T_{x\alpha} = T_{y\alpha} = T_{\perp\alpha}$ and $T_{z\alpha} = T_{\parallel\alpha}$. Replacing into Eqs. (14) and (15), we obtain

$$\begin{aligned}\langle E_iE_j^*\rangle &= 32i\pi^2k_B\sum_{\alpha}\frac{T_{j\alpha}T_{i\alpha}}{\omega T_{j\alpha} + \omega^*T_{i\alpha}}\\ &\quad\times\left[\lambda_{in}^{-1*}\chi_{nm}^{(\alpha)*}\Lambda_{mj}^{-1*} - \Lambda_{jm}^{-1}\chi_{mn}^{(\alpha)}\lambda_{ni}^{-1}\right].\end{aligned} \quad (20)$$

Equation (20) is valid for any general form of the susceptibilities and wave propagation provided F_{α} is a bi-Maxwellian. Notice that the sum over α behaves like a weighted temperature mean or effective temperature. The inverse of λ_{ij} can be evaluated through the Sherman-Morrison formula[40,41]

$$\lambda_{ij}^{-1} = \left(1 - \frac{c^2k^2}{\omega^2}\right)^{-1}\left(\delta_{ij} - \frac{c^2k_ik_j}{\omega^2}\right). \quad (21)$$

Although Eq. (20) is written in its general form, considerable simplifications can be achieved for the case of parallel wave propagation, namely $\mathbf{k} = k_{\parallel}\hat{z}$. Indeed, the susceptibility of each species can be written as a block diagonal tensor with $\chi_{xz}^{(\alpha)} = \chi_{yz}^{(\alpha)} = \chi_{zx}^{(\alpha)} = \chi_{zy}^{(\alpha)} = 0$ and[42,43]

$$\chi_{xx}^{(\alpha)} = \chi_{yy}^{(\alpha)} = \frac{1}{2}\left(\chi_{+}^{(\alpha)} + \chi_{-}^{(\alpha)}\right), \quad (22)$$

$$\chi_{xy}^{(\alpha)} = -\chi_{yx}^{(\alpha)} = \frac{i}{2}\left(\chi_{+}^{(\alpha)} - \chi_{-}^{(\alpha)}\right), \quad (23)$$

$$\chi_{zz}^{(\alpha)} = \frac{\omega_{p\alpha}^2}{2\pi k_{\parallel}^2u_{\parallel\alpha}^2}\left[1 + \xi_{\alpha}^{(0)}Z\left(\xi_{\alpha}^{(0)}\right)\right], \quad (24)$$

where

$$\chi_{\pm}^{(\alpha)} = \frac{\omega_{p\alpha}^2}{4\pi\omega^2}\left[A_{\alpha} + \left(\xi_{\alpha}^{(0)} + A_{\alpha}\xi_{\alpha}^{(\pm)}\right)Z\left(\xi_{\alpha}^{(\pm)}\right)\right]. \quad (25)$$

In Eqs. (22)–(25), $A_{\alpha} = u_{\perp\alpha}^2/u_{\parallel\alpha}^2 - 1$ is a measure of the thermal anisotropy. $\xi_{\alpha}^{(\sigma)} = (\omega - k_{\parallel}U_{\alpha} + \sigma\Omega_{\alpha})/(k_{\parallel}u_{\parallel\alpha})$ and $\sigma = \{0, \pm\}$. $\omega_{p\alpha} = \sqrt{4\pi n_{\alpha}q_{\alpha}^2/m_{\alpha}}$ and $\Omega_{\alpha} = q_{\alpha}B_0/m_{\alpha}c$ are the plasma and cyclotron frequencies of the species α, respectively. $Z(\xi)$ is the usual plasma dispersion function.[44]

For parallel propagation, λ_{ij} defined in Eq. (8) is a diagonal tensor. Then Λ_{ij} defined in Eq. (11) has a structure similar to that of $\chi_{ij}^{(\alpha)}$. The components of their inverses [needed in Eq. (20)] are given by

$$\lambda_{xx}^{-1} = \lambda_{yy}^{-1} = \frac{1}{1 - c^2k_{\parallel}^2/\omega^2}, \quad (26)$$

$$\lambda_{zz}^{-1} = 1, \quad (27)$$

$$\Lambda_{xx}^{-1} = \Lambda_{yy}^{-1} = \frac{\Lambda_{xx}}{\Lambda_{+}\Lambda_{-}}, \quad (28)$$

$$\Lambda_{xy}^{-1} = -\Lambda_{yx}^{-1} = -\frac{\Lambda_{xy}}{\Lambda_{+}\Lambda_{-}}, \quad (29)$$

$$\Lambda_{zz}^{-1} = \frac{1}{\Lambda_{zz}}, \quad (30)$$

while other components vanish, and where

$$\Lambda_\pm = \Lambda_{xx} \pm i\Lambda_{xy}. \tag{31}$$

After replacing Eqs. (22)–(31) into (20), we obtain for ω real

$$\langle |E_x|^2 \rangle = \langle |E_y|^2 \rangle = \frac{32\pi^2\omega}{\omega^2 - c^2k_\parallel^2} \sum_\alpha k_B T_{\perp\alpha} \mathrm{Im}\left[\frac{\Lambda_{xx}\chi_{xx}^{(\alpha)} + \Lambda_{xy}\chi_{xy}^{(\alpha)}}{\Lambda_+\Lambda_-}\right], \tag{32}$$

$$\langle E_x E_y^* \rangle = -\langle E_y E_x^* \rangle = -\frac{32\pi^2 i\omega}{\omega^2 - c^2k_\parallel^2} \sum_\alpha k_B T_{\perp\alpha} \mathrm{Re}\left[\frac{\Lambda_{xy}\chi_{xx}^{(\alpha)} - \Lambda_{xx}\chi_{xy}^{(\alpha)}}{\Lambda_+\Lambda_-}\right], \tag{33}$$

$$\langle |E_z|^2 \rangle = \frac{32\pi^2}{\omega} \sum_\alpha k_B T_{\parallel\alpha} \mathrm{Im}\left[\frac{\chi_{zz}^{(\alpha)}}{\Lambda_{zz}}\right]. \tag{34}$$

In the polarization coordinates, we define $E_\pm = (E_x \pm iE_y)/2$, so that

$$\langle |E_\pm|^2 \rangle = \frac{32\pi^2\omega}{\omega^2 - c^2k_\parallel^2} \sum_\alpha k_B T_{\perp\alpha} \mathrm{Im}\left[\frac{\chi_\pm^{(\alpha)}}{\Lambda_\pm}\right]. \tag{35}$$

Similarly, for the magnetic and density fluctuations, Eqs. (16) and (17) become

$$\langle |\rho|^2 \rangle = \frac{k_\parallel^2}{(4\pi)^2} \langle |E_z|^2 \rangle, \tag{36}$$

$$\langle |B_\pm|^2 \rangle = \frac{c^2k_\parallel^2}{\omega^2} \langle |E_\pm|^2 \rangle. \tag{37}$$

Equation (37) was studied by Navarro *et al.*[20] for anisotropic Maxwellian proton plasmas. For isotropic plasmas, Eqs. (32)–(37) reduce to the expressions considered by Sitenko,[4] Araneda *et al.*,[18] and Viñas *et al.*[19]

IV. NUMERICAL RESULTS FOR A CORE-BEAM PROTON PLASMA

The theory of plasma fluctuations described in Sec. II applies as long as plasma instabilities are weak or not present. For the solar wind at 1 AU, the threshold condition $\mathrm{Im}(\omega/\Omega_p) = 10^{-3}$ for the imaginary part of ω corresponds to timescales of the order of hours. This timescale is large enough for the plasma to be considered as quasi-stable. We will confine our attention to the zones constrained by this threshold for the instabilities.

For the case considered in Sec. III, the dispersion relation reduces to $\det(\Lambda_{ij}) = \Lambda_+\Lambda_-\Lambda_{zz} = 0$. Longitudinal and transverse waves are then represented by $\Lambda_{zz} = 0$ and $\Lambda_\pm = 0$, respectively. Since $\Lambda_-(k_\parallel, \omega) = \Lambda_+(-k_\parallel, -\omega^*)$, then the fluctuation spectrum for $\langle |E_+|^2 \rangle$ can be obtained by rotating the spectrum for $\langle |E_-|^2 \rangle$ in 180° around the origin $(k_\parallel, \omega) = (0, 0)$. We then evaluate the magnetic fluctuation spectrum $\langle |B_-|^2 \rangle$ and the dispersion relation $\Lambda_- = 0$ numerically for a plasma composed of a proton core ($\alpha = c$), a proton beam ($\alpha = b$), and an isotropic electron background ($\alpha = e$) which ensures the neutrality and current-free conditions

$$\sum_\alpha q_\alpha n_\alpha = 0, \tag{38}$$

$$\sum_\alpha q_\alpha n_\alpha U_\alpha = 0, \tag{39}$$

where q_α and n_α are the charge and density of species α, respectively. For simplicity, we use isotropic electrons with the same temperature as the proton core parallel temperature, namely, $T_{\perp e} = T_{\parallel e} = T_{\parallel c}$. The proton-to-electron mass ratio equals $m_p/m_e = 1836$ and we consider $v_A/c = 10^{-4}$ as the ratio between the Alfvén speed $v_A = B_0/\sqrt{4\pi n_p m_p}$ and the speed of light. In the reference frame of the proton core $U_c = 0$. All these quantities are constant throughout the paper.

In Fig. 1, we show the magnetic fluctuation spectrum $n_p\Omega_p\langle |B_-|^2 \rangle / 2B_0^2$, given by Eq. (37), for $u_{\parallel c}^2/v_A^2 = u_{\parallel b}^2/v_A^2 = 0.3$, $T_{\perp c}/T_{\parallel c} = T_{\perp b}/T_{\parallel b} = 0.1$, $U_b/v_A = 0.5$ and three values of $n_b/n_c = \{0, 0.1, 0.4\}$. The case $n_b/n_c = 0$ corresponds to no beam. We also show solutions for the

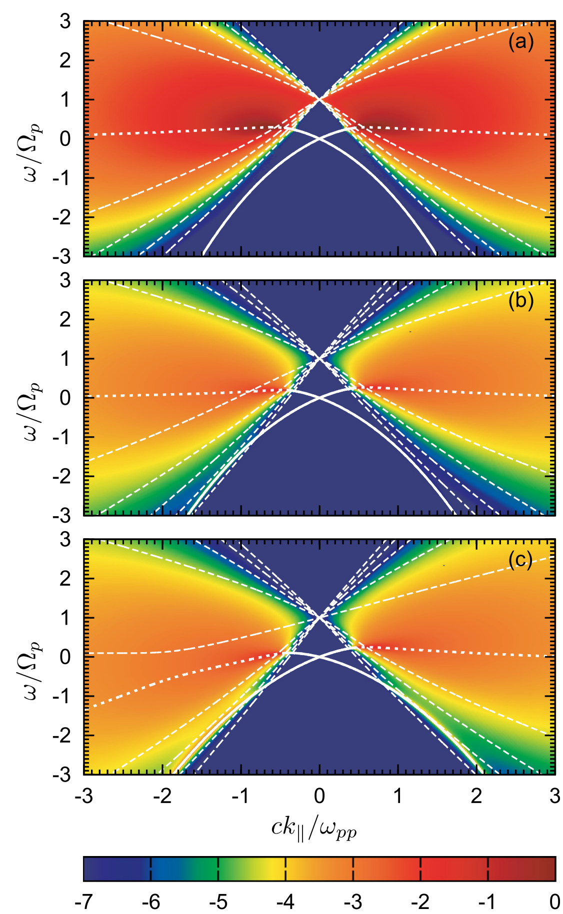

FIG. 1. Dimensionless transverse magnetic fluctuations $n_p\Omega_p\langle |B_-|^2 \rangle / 2B_0^2$, Eq. (37), and dispersion branches $\Lambda_- = 0$, Eq. (31), for $u_{\parallel c}^2/v_A^2 = u_{\parallel b}^2/v_A^2 = 0.3$, $T_{\perp c}/T_{\parallel c} = T_{\perp b}/T_{\parallel b} = 0.1$, $U_b/v_A = 0.5$ and (a) $n_b/n_c = 0$, (b) 0.1, and (c) 0.4. Solid lines are the Alfvén-cyclotron and fast modes. Dotted lines are frequencies for which the Alfvén-cyclotron mode is damped with $\mathrm{Im}(\omega/\Omega_p) < -0.001$. Dashed lines are the heavily damped modes or HOM. Logarithmic color scales are used.

dispersion relation for transverse waves $\Lambda_- = 0$. The curves crossing the origin are the Alfvén-cyclotron and fast modes. The set of curves crossing at $\omega/\Omega_p = 1$ are heavily damped modes of the dispersion relation known as Higher-Order Modes (HOM).[45] Since for a Maxwellian-like distribution there are an infinite number of HOMs, we only show the HOMs for which $\mathrm{Im}(\omega/\Omega_p) > -6.5$ at $ck_\parallel/\omega_{pp} = 3$.

In the case of no beam—e.g., Fig. 1(a)—the dispersion curves and fluctuation spectrum are symmetrical with respect to the frequency axis. However, these structures are no longer symmetric if we include a beam with $U_b \neq 0$. This asymmetry becomes more evident as n_b/n_c grows, as seen in Figs. 1(a)–1(c). In spite of this symmetry breaking, the fluctuation spectrum and the HOM curves share the same structure. The magnetic fluctuation level seems to be enhanced between the two least damped HOMs in Figs. 1(a)–1(c). This feature agrees with previous results.[18–20]

Because of the condition $\Lambda_- = 0$ [see Eqs. (32)–(35)], the electromagnetic fluctuation level exhibits intense peaks near the modes with $\mathrm{Im}(\omega) \sim 0$. Here, the collective effects of the plasma take place to enhance the spontaneous fluctuations. Hence, one can expect that the normal modes could be excited for some values of the macroscopic parameters if sufficient free energy, say a thermal anisotropy or a relative drift between species, is available.

In Figs. 1(b) and 1(c), a weak instability is present in the fast mode in the third and fourth quadrants, with $\mathrm{Im}(\omega/\Omega_p) < 10^{-4}$. The maximum growth-rate occurs for modes whose phase-speeds $\omega/k_\parallel$ have the same sign as U_b. For $\omega/k_\parallel U_b > 0$, a large portion of the beam VDF interacts with these waves (a resonant instability), whereas for $\omega/k_\parallel U_b < 0$ the wave resonates with the tail of the VDF (a non-resonant instability).[46] This means that the fluctuation level will increase near the resonant unstable wave more than around the non-resonant one. In Figs. 1(a)–1(c), we observe that the fluctuation intensity increases near the fast modes (third and fourth quadrants) as n_b/n_c increases, but the magnetic fluctuation level concentrates in the third quadrant due to the asymmetry caused by U_b. Larger values for the free energy will drive an instability of the fast modes and, consequently, the electromagnetic fluctuations will enhance near these modes. Further studies on ion-beam instabilities can be found in the literature.[47,48]

The presence of drifting protons reveals another characteristic concerning the HOMs. As n_b/n_c increases, the HOM with the smallest positive slope interacts with the Alfvén-cyclotron branch until a gap appears for $n_b/n_c \sim 0.291$ near $ck_\parallel/\omega_{pp} \sim -1.75$ and $\omega/\Omega_p \sim -1.35$ [see transition between Figs. 1(b) and 1(c)]. Similar characteristics are observed for plasmas of protons and drifting alpha particles.[45] Unlike the other HOMs, this mode is a branch of the dispersion relation in the limit of zero temperature, but is damped at larger temperatures. We will see that electromagnetic fluctuations will increase near this mode for some values of the macroscopic values.

In Fig. 2, we show the magnetic spectrum for fixed values of $n_b/n_c = 0.3$ and $U_b/v_A = 0.5$ for different values of the anisotropy $T_{\perp c}/T_{\parallel c} = T_{\perp b}/T_{\parallel b}$. For the parameters

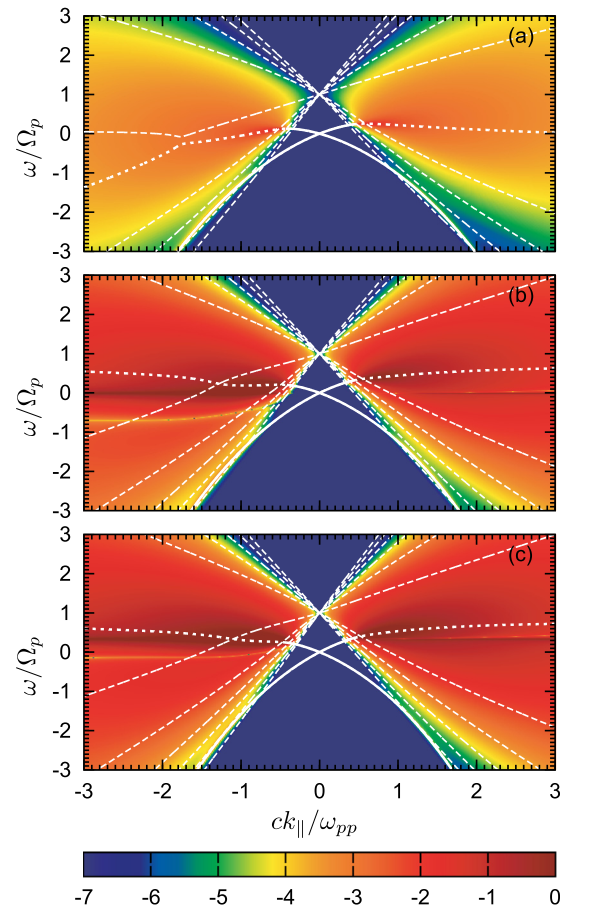

FIG. 2. Same as Fig. 1, but for $n_b/n_c = 0.3$, $U_b/v_A = 0.5$ and (a) $T_{\perp c}/T_{\parallel c} = T_{\perp b}/T_{\parallel b} = 0.1$, (b) 1.0, and (c) 1.5.

used in Figs. 2(a) and 2(c), firehose and proton-cyclotron instabilities develop, respectively, with $\mathrm{Im}(\omega/\Omega_p) < 4 \times 10^{-4}$ in both cases. Panel (b) corresponds to thermal equilibrium. The magnetic fluctuating intensity increases near the Alfvén-cyclotron branch as the anisotropy increases. Both the fluctuation and HOM structure show no qualitative changes.

In Fig. 3, we show the influence of the proton beam parallel temperature. For smaller values of $u^2_{\parallel b}/v^2_A$, there is a group of HOMs whose damping decreases in Fig. 3, and they fill the empty space inside the HOMs of Figs. 1 and 2. As $u^2_{\parallel b}/v^2_A$ increases beyond the values of Figs. 1 and 2, these HOMs separate from each other, and converge to a structure that is similar to the ones presented in Figs. 1 and 2.

In Fig. 4, we show the magnetic fluctuation spectrum for a lower value of the parallel temperatures $u^2_{\parallel b}/v^2_A = u^2_{\parallel c}/v^2_A = 0.03$, and different values of U_b. The asymmetry of the HOMs and the fluctuation structure with respect to the vertical axis is even more evident than in Figs. 1–3. In fact, the fluctuation structure follows the direction of $\omega = \Omega_p + k_\parallel U_\alpha$. There is a group of HOMs which seem to bound each of the fluctuation structures [two in Fig. 4(a) and four in Figs. 4(b) and 4(c)]. In addition to the magnetic fluctuations near the Alfvén-cyclotron mode, there exists an enhanced fluctuation structure near $\omega \sim \Omega_p$, and $k_\parallel < 0$, which turns out to be associated with the HOM with the least damping. This HOM

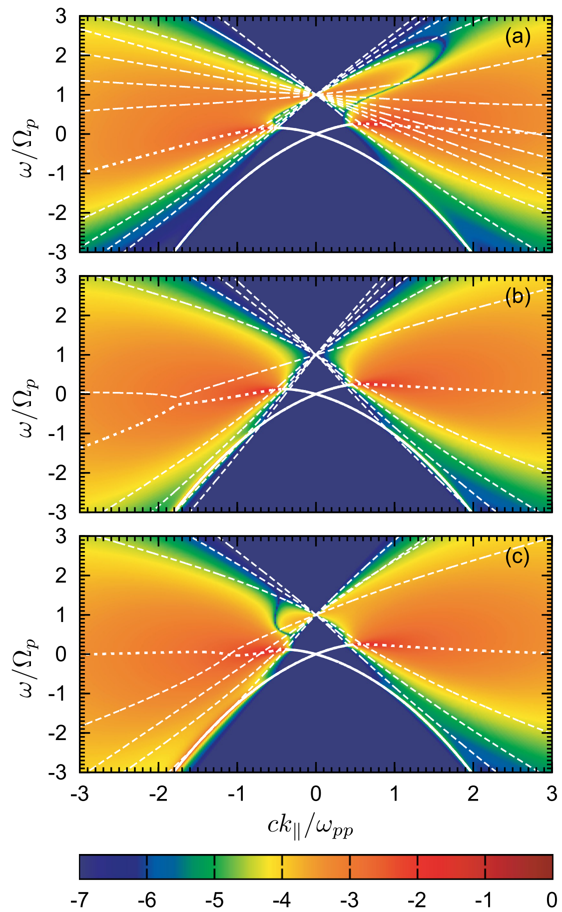

FIG. 3. Same as Fig. 2, but for $T_{\perp c}/T_{\parallel c} = T_{\perp b}/T_{\parallel b} = 0.1$ and (a) $u_{\parallel b}^2/v_A^2 = 0.1$, (b) 0.3, and (c) 0.4.

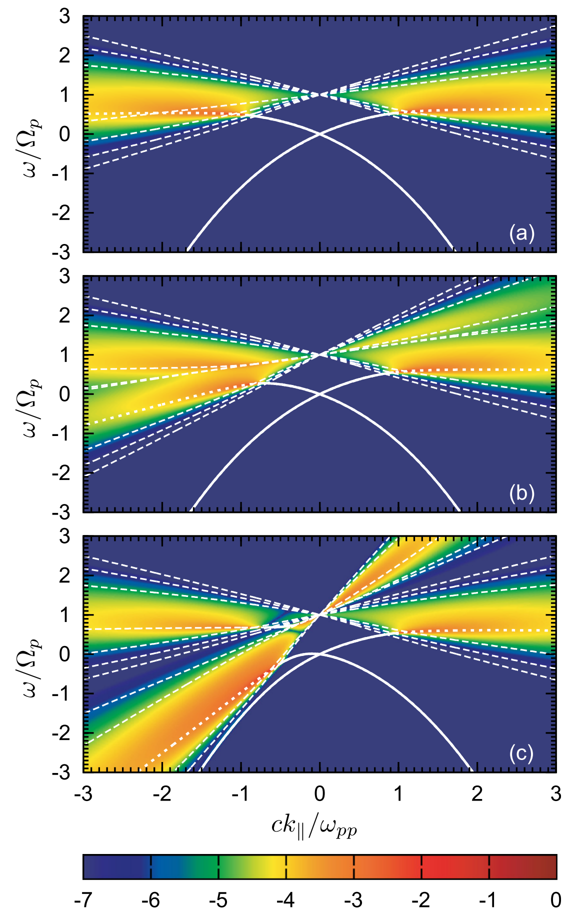

FIG. 4. Same as Fig. 3, but for $u_{\parallel b}^2/v_A^2 = u_{\parallel c}^2/v_A^2 = 0.03$ and (a) $U_b/v_A = 0.1$, (b) 0.5, and (c) 1.5.

corresponds to the branch of the fluid-like description crossing at $\omega = \Omega_p$. Since the fluctuation level increases near this mode, we could expect that this mode would become unstable if there is enough free energy available to excite it.

In Fig. 4(c), the fast mode in the third quadrant is unstable with $\mathrm{Im}(\omega/\Omega_p) < 10^{-4}$. As the drift U_b increases, this mode becomes unstable and the magnetic fluctuation is enhanced around it. For lower values of the temperatures, the fluctuation structure is narrower than that shown in Fig. 4, thus larger values of U_b are necessary to excite the fast mode.

V. SUMMARY

We have derived analytically the electromagnetic fluctuations in plasmas near equilibrium as a function of an arbitrary velocity distribution function. Although there exist similar works aimed to its derivation in unmagnetized plasmas,[12] here we provide a numerically simpler way to analyze these fluctuations without the use of integral representations (form factors). Furthermore, our general expressions apply to magnetized and thermally anisotropic plasmas near their stationary state. They are also determined by the structure of the velocity distribution function and the susceptibility of each species.

We applied the general formalism to the special case of a plasma with a magnetized proton core and beam. In general, the fluctuation spectrum is enhanced near the normal modes of the system with $\mathrm{Im}(\omega) \sim 0$. Any instability in the normal branches of the dispersion relation, including the HOMs, could emerge within the magnetic fluctuations if free energy is available. Furthermore, the magnetic fluctuations appear to grow as the plasma approaches the instability thresholds from a quasi-stable state. The structure of magnetic fluctuations seems to share those of the heavily damped modes of the dispersion relation even for drifting species. For some macroscopic plasma parameters, the fluctuation spectrum seems to be constrained by the least damped modes. This suggests that heavily damped modes play a fundamental role in the emission and re-absorption of thermally induced magnetic fluctuations in plasmas.

For the special case of electromagnetic fluctuations propagating parallel to an ambient magnetic field in isotropic Maxwellian-like plasmas, the general form of the electromagnetic spectrum reduces to results considered in earlier works.[4,18–20] The parallel propagating thermally induced magnetic fluctuations seem to be a relevant contribution to the solar wind plasma under conditions stable to anisotropy-driven instabilities.[20] However, we expect that fluctuations with $k_\perp \neq 0$ should also provide a significant contribution since the random motion of particles should also produce fluctuations that propagate obliquely with respect to the background magnetic field. Thus, this and earlier studies[18–20] should be regarded as a first step towards a more general analysis including nonparallel propagating fluctuations, which will be considered in a follow-up paper.

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[1]H. B. Callen and T. Welton, Phys. Rev. **83**, 34 (1951).
[2]V. P. Silin, Radiofizika (U.S.S.R.) **2**, 198 (1959).
[3]S. Ichimaru, Ann. Phys. **20**, 78 (1962).
[4]A. G. Sitenko, *Electromagnetic Fluctuations in Plasma* (Academic, New York, 1967).
[5]D. A. Chapman and D. O. Gericke, Phys. Rev. Lett. **107**, 165004 (2011).
[6]B. Li and R. D. Hazeltine, Phys. Rev. E **73**, 065402 (2006).
[7]N. Meyer-Vernet, P. Couturier, S. Hoang, J. L. Steinberg, and R. D. Zwickl, J. Geophys. Res. **91**, 3294, doi:10.1029/JA091iA03p03294 (1986).
[8]E. J. Lund, J. LaBelle, and R. A. Treumann, J. Geophys. Res. **99**, 23651, doi:10.1029/94JA02134 (1994).
[9]N. Meyer-Vernet, S. Hoang, K. Issautier, M. Moncuquet, and G. Marcos, *Plasma Thermal Noise: The Long Wavelength Radio Limit* (American Geophysical Union, 2013), pp. 67–74.
[10]K. Issautier, M. Moncuquet, N. Meyer-Vernet, and S. Hoang, Astrophys. Space Sci. **277**, 309 (2001).
[11]M. Moncuquet, A. Lecacheux, N. Meyer-Vernet, B. Cecconi, and W. S. Kurth, Geophys. Res. Lett. **32**, L20S02, doi:10.1029/2005GL022508 (2005).
[12]R. Schlickeiser and P. H. Yoon, Phys. Plasmas **19**, 022105 (2012).
[13]M. Lazar, P. H. Yoon, and R. Schlickeiser, Phys. Plasmas **19**, 122108 (2012).
[14]T. Felten, R. Schlickeiser, P. H. Yoon, and M. Lazar, Phys. Plasmas **20**, 052113 (2013).
[15]T. Felten and R. Schlickeiser, Phys. Plasmas **20**, 082116 (2013).
[16]T. Felten and R. Schlickeiser, Phys. Plasmas **20**, 082117 (2013).
[17]E. J. Lund, R. A. Treumann, and J. LaBelle, Phys. Plasmas **3**, 1234 (1996).
[18]J. A. Araneda, H. Astudillo, and E. Marsch, Space Sci. Rev. **172**, 361 (2012).
[19]A. F. Viñas, P. S. Moya, R. Navarro, and J. A. Araneda, Phys. Plasmas **21**, 012902 (2014).
[20]R. E. Navarro, P. S. Moya, V. Muñoz, J. A. Araneda, A. F.-Viñas, and J. A. Valdivia, Phys. Rev. Lett. **112**, 245001 (2014).
[21]S. Bale, J. Kasper, G. Howes, E. Quataert, C. Salem, and D. Sundkvist, Phys. Rev. Lett. **103**, 211101 (2009).
[22]S. P. Gary, M. E. McKean, D. Winske, B. J. Anderson, R. E. Denton, and S. A. Fuselier, J. Geophys. Res. **99**, 5903, doi:10.1029/93JA03583 (1994).
[23]S. P. Gary, H. Li, S. O'Rourke, and D. Winske, J. Geophys. Res. **103**, 14567, doi:10.1029/98JA01174 (1998).
[24]J. C. Kasper, A. J. Lazarus, and S. P. Gary, Geophys. Res. Lett. **29**, 1839, doi:10.1029/2002GL015128 (2002).
[25]P. Hellinger, P. Trávníček, J. C. Kasper, and A. J. Lazarus, Geophys. Res. Lett. **33**, L09101, doi:10.1029/2006GL025925 (2006).
[26]E. Marsch, K.-H. Mühlhäuser, R. Schwenn, H. Rosenbauer, W. Pilipp, and F. M. Neubauer, J. Geophys. Res. **87**, 52, doi:10.1029/JA087iA01p00052 (1982).
[27]E. Marsch and S. Livi, J. Geophys. Res. **92**, 7263, doi:10.1029/JA092iA07p07263 (1987).
[28]B. E. Goldstein, M. Neugebauer, L. D. Zhang, and S. P. Gary, Geophys. Res. Lett. **27**, 53, doi:10.1029/1999GL003637 (2000).
[29]E. Marsch, Space Sci. Rev. **172**, 23 (2012).
[30]E. Marsch, K.-H. Mühlhäuser, H. Rosenbauer, R. Schwenn, and F. M. Neubauer, J. Geophys. Res. **87**, 35, doi:10.1029/JA087iA01p00035 (1982).
[31]S. Olbert, in *Physics of the Magnetosphere*, Astrophysics and Space Science Library Vol. 10, edited by R. D. L. Carovillano and J. F. McClay (Springer, 1968), p. 641.
[32]V. M. Vasyliunas, J. Geophys. Res. **73**, 2839, doi:10.1029/JA073i009p02839 (1968).
[33]W. C. Feldman, J. R. Asbridge, S. J. Bame, and M. D. Montgomery, J. Geophys. Res. **78**, 2017, doi:10.1029/JA078i013p02017 (1973).
[34]V. Formisano, G. Moreno, F. Palmiotto, and P. C. Hedgecock, J. Geophys. Res. **78**, 3714, doi:10.1029/JA078i019p03714 (1973).
[35]M. Lockwood, B. J. I. Bromage, R. B. Horne, J.-P. St-Maurice, D. M. Willis, and S. W. H. Cowley, Geophys. Res. Lett. **14**, 111, doi:10.1029/GL014i002p00111 (1987).
[36]M. R. Collier, D. C. Hamilton, G. Gloeckler, P. Bochsler, and R. B. Sheldon, Geophys. Res. Lett. **23**, 1191, doi:10.1029/96GL00621 (1996).
[37]K. Chotoo, M. R. Collier, A. B. Galvin, D. C. Hamilton, and G. Gloeckler, J. Geophys. Res. **103**, 17441, doi:10.1029/98JA01173 (1998).
[38]K. Chotoo, N. A. Schwadron, G. M. Mason, T. H. Zurbuchen, G. Gloeckler, A. Posner, L. A. Fisk, A. B. Galvin, D. C. Hamilton, and M. R. Collier, J. Geophys. Res. **105**, 23107, doi:10.1029/1998JA000015 (2000).
[39]D. Chandler, *Introduction to Modern Statistical Mechanics* (Oxford Univ. Press, Oxford, 1987), p. 252.
[40]J. Sherman and W. J. Morrison, Ann. Math. Stat. **21**, 124 (1950).
[41]M. S. Bartlett, Ann. Math. Stat. **22**, 107 (1951).
[42]D. G. Swanson, *Plasma Waves* (Academic, San Diego, 1989).
[43]T. H. Stix, *Waves in Plasmas* (AIP, New York, 1992).
[44]B. D. Fried and S. D. Conte, *The Plasma Dispersion Function* (Academic, San Diego, California, 1961).
[45]H. F. Astudillo, J. Geophys. Res. **101**, 24433, doi:10.1029/96JA01586 (1996).
[46]S. Gary, Space Sci. Rev. **56**, 373 (1991).
[47]S. P. Gary, *Theory of Space Plasma Microinstabilities* (Cambridge University Press, New York, 1993).
[48]W. Daughton and S. P. Gary, J. Geophys. Res. **103**, 20613, doi:10.1029/98JA01385 (1998).